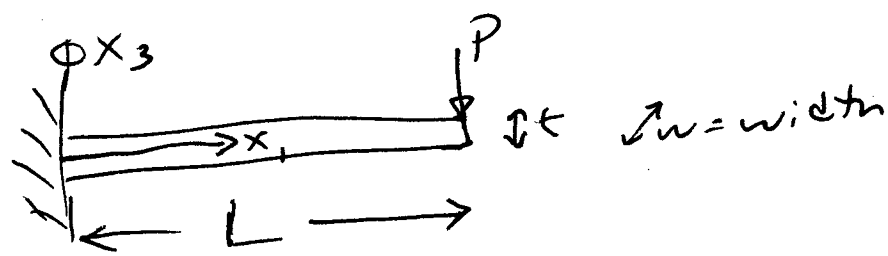
PAL
4/24/08

Unified Engineering Problem Set
Week 13 Spring, 2008

SOLUTIONS

M13.1

Cantilevered beam:



(a) The basic equation for energy per unit volume is:

$$u = \int_0^{\epsilon} \sigma \, \delta\epsilon$$

Prior to yielding (and assuming linear behavior):

$$\sigma = E\epsilon$$

Combine these two expressions to get:

$$u = \int_0^{\epsilon} E\epsilon \, \delta\epsilon$$

$$\Rightarrow u = \frac{1}{2} E \epsilon^2 \Big]_0^{\epsilon} = \frac{1}{2} E \epsilon^2$$

Place this in terms of σ and E using the stress-strain relation in the form:

$$\epsilon = \frac{\sigma}{E}.$$

This gives:

$$u = \frac{1}{2} E \left(\frac{\sigma}{E}\right)^2$$

$$\Rightarrow u = \frac{1}{2} \frac{\sigma^2}{E} \qquad (1)$$

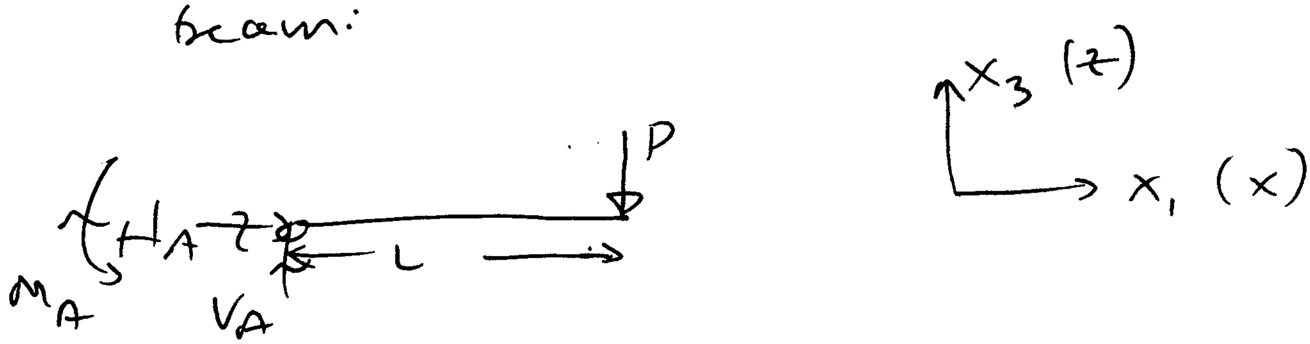
(per unit volume)

For a beam, recall that:

$$\sigma = -\frac{Mz}{I} \qquad (2)$$

So need an expression for the moment, M, in the beam.

→ Draw the free body diagram of the beam:

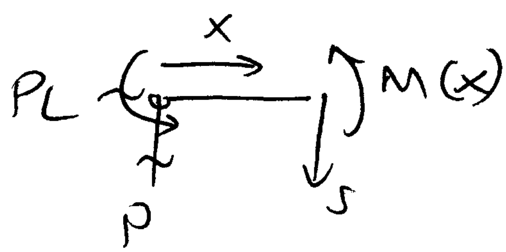


$$\Sigma F_{x_1} = 0 \Rightarrow H_A = 0$$

$$\Sigma F_{x_3} = 0 \quad \uparrow + \Rightarrow V_A - P = 0 \Rightarrow V_A = P$$

$$\Sigma M_A = 0 \quad \circlearrowleft + \Rightarrow M_A - PL = 0 \Rightarrow M_A = PL$$

Now cut the beam:



$$\Sigma M_x = 0 \quad \circlearrowleft + \Rightarrow M(x) + PL - Px = 0$$

$$\Rightarrow M(x) = P(x-L)$$

Use this in the expression for stress (equation (2)) to get:

$$\sigma = -\frac{P(x-L)z}{I} \qquad (3)$$

and then in the expression for energy of equation (1):

$$\underset{\text{(per unit volume)}}{U} = \frac{1}{2}\frac{\left(-\frac{P(x-L)z}{I}\right)^2}{E}$$

To get to the expression for the total energy, this need to be integrated over the volume of the beam:

$$U_{total} = \iiint u \, dV = \iiint u \, dx \, dy \, dz$$

with $x = 0$ to L
$y = -w/2$ to $w/2$
$z = -t/2$ to $t/2$

so:

$$U_{total} = \int_x \int_y \int_z \frac{P^2(x^2 - 2xL + L^2) z^2}{2EI^2} \, dx \, dy \, dz$$

All items except x and z are constants.
so:

$$U_{total} = \frac{P^2}{2EI^2} \int_x \int_y \int_z (x^2 - 2xL + L^2) z^2 \, dx \, dy \, dz$$

integrate in z:

$$= \frac{P^2}{2EI^2} \int_x \int_y (x^2 - 2xL + L^2) \, dx \, dy \left. \frac{z^3}{3} \right]_{-t/2}^{t/2}$$

t is a constant with respect to x and y. This comes out of the integral with

$$\left. \frac{z^3}{3} \right]_{-t/2}^{t/2} = 2 \cdot \frac{t^3}{24} = \frac{t^3}{12}$$

integrate in y:

$$U_{total} = \frac{P^2 t^3}{24 E I^2} \int_x (x^2 - 2xL + L^2)\, dx \; \underbrace{y \Big]_{-w/2}^{w/2}}_{=w}$$

finally integrating in x:

$$U_{total} = \frac{P^2 w t^3}{24 E I^2} \left[\frac{x^3}{3} - 2\frac{x^2 L}{2} + L^2 x \right]_0^L$$

$$= \frac{P^2 w t^3}{24 E I^2} \left[\frac{L^3}{3} - L^3 + L^3 \right]$$

$$= \frac{P^2 w t^3 L^3}{72 E I^2}$$

Now recall that $I = \frac{bh^3}{12} = \frac{wt^3}{12}$

substituting this in the expression for U_{total} gives:

$$U_{total} = \frac{P^2 w t^3 L^3}{72 E \left(\frac{wt^3}{12}\right)^2}$$

$$= \frac{144 P^2 w t^3 L^3}{72 E w^2 t^6}$$

and finally:

$$\boxed{U_{total} = \frac{2 P^2 L^3}{E w t^3}} \qquad (4)$$

CHECK THE UNITS!

Total Energy: $[F \cdot L] \stackrel{?}{=} \dfrac{[F]^2 [L]^3}{[F/L^2][L][L^3]}$

$$\stackrel{?}{=} \frac{[F][L]^3}{[L]^2}$$

$$\stackrel{?}{=} [F \cdot L] \quad \checkmark$$

YES

(b) First consider the issue of yielding and work the expression for U_{total} to include the material parameter of yield stress $= \sigma_Y$

Recall that: $\sigma = -\dfrac{Mz}{I}$

and with the expression for moment that stress is expressed in equation (3) as:

$$\sigma = -\frac{P(x-L)z}{I}$$

Find the maximum value of stress in the beam and set this to σ_Y to determine the maximum possible value of load (as dependent on the material and σ_Y).

The maximum value occurs for

$$x = 0$$
$$z = + \text{or} - t/2$$

This is generally tensile yielding, so:

$$\sigma_{max} = \sigma_Y = \frac{PLt}{2I}$$

using $I = \frac{wt^3}{12}$ and solving for P_{max}:

$$P_{max} = \frac{\sigma_Y\, w t^2}{6\cancel{L}}$$

use this in the expression of equation (4) for U_{total}:

$$U_{total} = \frac{2\left(\frac{\sigma_Y w t^2}{6L}\right)^2 L^3}{E w t^3}$$

to get:

$$U_{total} = \frac{2\sigma_Y^2 w^2 t^4 L^3}{36 L^2 E w t^3}$$

$$\Rightarrow U_{total} = \frac{\sigma_Y^2 w t L}{18 E} \qquad (5)$$

again, CHECK UNITS:

$$[F \cdot L] \overset{?}{=} \frac{[F/L^2]^2 [L][L][L]}{[F/L^2]} \overset{?}{=} [F \cdot L]$$

✓ Yes

Proceeding....

(i) for a given thickness t

w and L are constants

t is for this case

$$\Rightarrow \quad \text{Energy} = \frac{\sigma_Y^2}{E} \cdot \text{constant}$$

$$\text{so} \quad \boxed{\text{maximize } \frac{\sigma_Y^2}{E}}$$

(ii) for a given mass of material

$\Rightarrow$ volume · density = constant = total mass

so $WLt \cdot \rho$ = constant

material parameter to include

w and L are also constants giving:

$$t \cdot \rho = \text{constant}$$

Look at the expression of equation (5) for U_{total}. All items are constants except for σ_Y, E and t. Thus:

$$U_{total} = \frac{\sigma_Y^2 t}{E} \cdot \text{constant}$$

Since $t \cdot \rho = \text{constant}$

$$\Rightarrow t = \frac{\text{constant}}{\rho}$$

and placing this in the derived expression for U_{total} gives:

$$U_{total} = \frac{\sigma_Y^2}{E\rho} \cdot \text{constant}$$

$$\Rightarrow \boxed{\text{maximize } \frac{\sigma_Y^2}{E\rho}}$$

(iii) for a given cost of material

The key material parameter to include is the: cost per mass = c

Total cost = (cost per mass)(mass) = constant

so: $\underset{\text{(volume)}}{wLt} \cdot \underset{\text{(density)}}{\rho} \cdot \underset{\text{(cost/mass)}}{c} = \text{constant}$

Again, w and L are constants so:

$$t \rho c = \text{constant}$$

As in (ii), all items in the expression of equation (5) for U_{total} are constants such that:

$$U_{total} = \frac{\sigma_Y^2 t}{E} \cdot \text{constant}$$

since $t \rho C = \text{constant}$

$$\Rightarrow t = \frac{\text{constant}}{\rho C}$$

and placing this in the derived expression for U_{total} gives:

$$U_{total} = \frac{\sigma_Y^2}{E \rho C} \cdot \text{constant}$$

$$\Rightarrow \boxed{\text{maximize } \frac{\sigma_Y^2}{E \rho C}}$$

(c) Looking at these six materials, calculate the pertinent combinations of parameters (i.e. the criterion for each case) and compare:

Material	Given thickness: maximize (σ_Y^2/E) $[10^6\ Pa]$	Given Mass: maximize $(\sigma_Y^2/E\rho)$ $[Pa/(g/m^3)]$	Given Cost: maximize $(\sigma_Y^2/E\rho c)$ $\left[\frac{10^3\ N\cdot m}{\$}\right]$
Al alloy	3.57	1.32	0.53
Spring steel	$\boxed{27.4}$	3.43	$\boxed{0.98}$
Wood	0.49	0.98	$\boxed{0.98}$
Titanium	16.9	$\boxed{3.76}$	0.29
Glass/Epoxy	1.14	0.57	0.048
Graphite/Epoxy	4.23	2.82	0.014

NOTE: Being clear and consistent on units is important. Look for each case.

~~THI~~CKNESS: $\dfrac{\sigma_Y^2}{E} = \dfrac{[10^6\ Pa]^2}{[10^6\ Pa]} = [10^3\ Pa]$ extra 10^3 Pa comes from numbers

MASS: $\dfrac{\sigma_Y^2}{E\rho} = \dfrac{[10^6\ Pa]^2}{[10^6\ Pa]\,[g/m^3]} = [Pa/(g/m^3)]$

could also get to: $[N/m^2/(g/m^3)] = \left[\dfrac{N\cdot m}{g}\right]$

COST: $\dfrac{\text{Mass result}}{c} = \left[\dfrac{N\cdot m}{g}\right]\cdot\dfrac{1}{[\$/10^3\ g]}$

$= \left[\dfrac{10^3\ N\cdot m}{\$}\right]$

or could be $[10^3\ Pa\cdot m^3/\$]$

Comments

- For the thickness criterion, spring steel is best material by nearly a factor of 2
- For the mass criterion, titanium just beats out spring steel
- For the cost criterion, wood ties spring steel
- Thought: Does wood get used in many home applications because of cost issues primarily? Consider a diving board.

14.2 <u>Condition A</u>: $\sigma_{11} = 4q \quad \sigma_{12} = 0$

$\sigma_{22} = -2q \quad \sigma_{13} = 0$

$\sigma_{33} = q \quad \sigma_{23} = 0$

<u>Condition B</u>: $\sigma_{11} = q \quad \sigma_{12} = 2q$

$\sigma_{22} = 4q \quad \sigma_{13} = 0$

$\sigma_{33} = 0.5q \quad \sigma_{23} = 0$

<u>Condition C</u>: $\sigma_{11} = -3q \quad \sigma_{12} = 0$

$\sigma_{22} = -3q \quad \sigma_{13} = 0$

$\sigma_{33} = -3q \quad \sigma_{23} = 0$

<u>Condition D</u>: $\sigma_{11} = q \quad \sigma_{12} = 0$

$\sigma_{22} = 2q \quad \sigma_{13} = 0$

$\sigma_{33} = 4q \quad \sigma_{23} = 0$

$\sigma_{yield} = 300$ MPa

(a) Application of the Tresca condition requires knowledge of the principal stresses.

→ For conditions A, C, and D, there are no applied shear stresses, so the applied normal stresses are the principal stresses.

Put these in appropriate order based on magnitude:

Condition A	Condition C	Condition D
$\sigma_I = \sigma_{11} = 4q$	$\sigma_I = \sigma_{11} = -3q$	$\sigma_I = \sigma_{33} = 4q$
$\sigma_{II} = \sigma_{22} = -2q$	$\sigma_{II} = \sigma_{22} = -3q$	$\sigma_{II} = \sigma_{22} = 2q$
$\sigma_{III} = \sigma_{33} = q$	$\sigma_{III} = \sigma_{33} = -3q$	$\sigma_{III} = \sigma_{11} = q$

→ For condition B, there is no applied shear stress in the 3-axis since $\sigma_{13} = \sigma_{23} = 0$ so σ_{33} is a principal stress. However, σ_{12} is nonzero, so the principal stresses in the 1-2 plane need to be determined.

(from last term) for plane stress the principal stresses are the roots of the equation:

$$\tau^2 - \tau(\sigma_{11} + \sigma_{22}) + (\sigma_{11}\sigma_{22} - \sigma_{12}^2) = 0$$

For condition B ⇒

$$\tau^2 - \tau(q + 4q) + ([q][4q] - [2q]^2) = 0$$

$$\Rightarrow \tau^2 - 5q\tau + (4q^2 - 4q^2) = 0$$

$$\Rightarrow \tau(\tau - 5q) = 0$$

giving: $\tau = \sigma_I = 5q$

$\tau = \sigma_{II} = 0$

Finally in order for Condition B

$$\sigma_I = 5q$$
$$\sigma_{II} = 0.5q$$
$$\sigma_{III} = 0$$

$\longrightarrow$ Now apply the Tresca criterion where yields occurs if:

$$|\sigma_I - \sigma_{II}| = \sigma_Y$$

or

$$|\sigma_{II} - \sigma_{III}| = \sigma_Y$$

or

$$|\sigma_{III} - \sigma_I| = \sigma_Y$$

in addition, the directionality associated with this is that yielding occurs via shear on the plane of maximum shear stress corresponding to the difference in those two principal stresses.

Apply each condition....

Condition A: $|\sigma_I - \sigma_{II}| = |4q - (-2q)| = \sigma_Y$

$\Rightarrow 6q = 300$ MPa

$\Rightarrow q = 50$ MPa

$|\sigma_{II} - \sigma_{III}| = |-2q - q| = \sigma_Y$

$\Rightarrow 3q = 300$ MPa

$\Rightarrow q = 100$ MPa

$|\sigma_{III} - \sigma_I| = |q - 4q| = \sigma_Y$

$\Rightarrow 3q = 300$ MPa

$\Rightarrow q = 100$ MPa

Critical case is first (A)

$\Rightarrow$ yielding at $q = 50$ MPa on plane at 45° between σ_{11} and σ_{22}

Condition B: $|\sigma_I - \sigma_{II}| = |5q - 0.5q| = \sigma_Y$

$\Rightarrow 4.5q = 300$ MPa

$\Rightarrow q = 66.7$ MPa

$|\sigma_{II} - \sigma_{III}| = |0.5q - 0| = \sigma_Y$

$\Rightarrow 0.5q = 300$ MPa

$\Rightarrow q = 600$ MPa

$|\sigma_{III} - \sigma_I| = |0 - 5q| = \sigma_Y$

$\Rightarrow 5q = 300$ MPa

$\Rightarrow q = 60$ MPa

critical case is ~~third~~ B

$\Rightarrow$ $\boxed{\text{yielding at } q = 60\,\text{MPa on plane at } 45^\circ \text{ to direction of principal stress* in 1-2 plane and } \sigma_{33}}$

* find this angle in 1-2 plane by using:

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{12}}{\sigma_{11} - \sigma_{22}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{4q}{q - (-4q)}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(4/5\right)$$

$$\Rightarrow \theta_p = \frac{1}{2}(38.7^\circ) = 19.3^\circ$$

Condition C: $|\sigma_I - \sigma_{II}| = 0 \; \ldots$

All differences are 0 since this is hydrostatic stress C

$\Rightarrow$ $\boxed{\text{No yielding}}$

Condition D: $|\sigma_I - \sigma_{II}| = |4q - 2q| = \sigma_Y$

$$\Rightarrow 2q = 300\,\text{MPa}$$

$$\Rightarrow q = 150\,\text{MPa}$$

$$|\sigma_{II} - \sigma_{III}| = |2q - q| = \sigma_Y$$
$$\Rightarrow \quad q = 300\,MPa$$
$$|\sigma_{III} - \sigma_I| = |q - 4q| = \sigma_Y$$
$$\Rightarrow 3q = 300\,MPa$$
$$\Rightarrow \quad q = 100\,MPa$$

critical case is third D

$$\Rightarrow \boxed{\text{yielding at } q = 100\,MPa \text{ on plane at } 45° \text{ between } \sigma_{11} \text{ and } \sigma_{33}}$$

(5) The von Mises criterion is:

$$(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 = 2\sigma_Y^2$$

Look at each condition again:

<u>Condition A</u>:

$$(4q - (-2q))^2 + (-2q - q)^2 + (q - 4q)^2 = 2\sigma_Y^2$$
$$\Rightarrow 36q^2 + 9q^2 + 9q^2 = 2\sigma_Y^2$$
$$q = \sqrt{\frac{2}{54}}\,\sigma_Y$$
$$\Rightarrow \text{for A} \quad \boxed{q = 57.7\,MPa}$$

Condition B:

$$(5q - 0.5q)^2 + (0.5q - 0)^2 + (0 - 5q)^2 = 2\sigma_Y^2$$

$$\Rightarrow 20.25q^2 + 0.25q^2 + 25q^2 = 2\sigma_Y^2$$

$$\Rightarrow 45.5q^2 = 2\sigma_Y^2$$

$$\Rightarrow q = \sqrt{\frac{2}{45.5}}\,\sigma_Y$$

for (D): $\boxed{q = 62.9\,MPa}$

Condition C:

$$(-3q - (-3q))^2 + (-3q - (-3q))^2 + (-3q - (3q))^2 = 2\sigma_Y^2$$

WAIT! Again all the differences are 0 as this is hydrostatic stress

$\Rightarrow$ $\boxed{\text{NO Yielding}}$

Condition D:

$$(4q - 2q)^2 + (2q - q)^2 + (q - 4q)^2 = 2\sigma_Y^2$$

$$\Rightarrow 4q^2 + q^2 + 9q^2 = 2\sigma_Y^2$$

$$\Rightarrow q = \sqrt{\frac{2}{14}}\,\sigma_Y^2$$

for (D): $\boxed{q = 113.4\,MPa}$

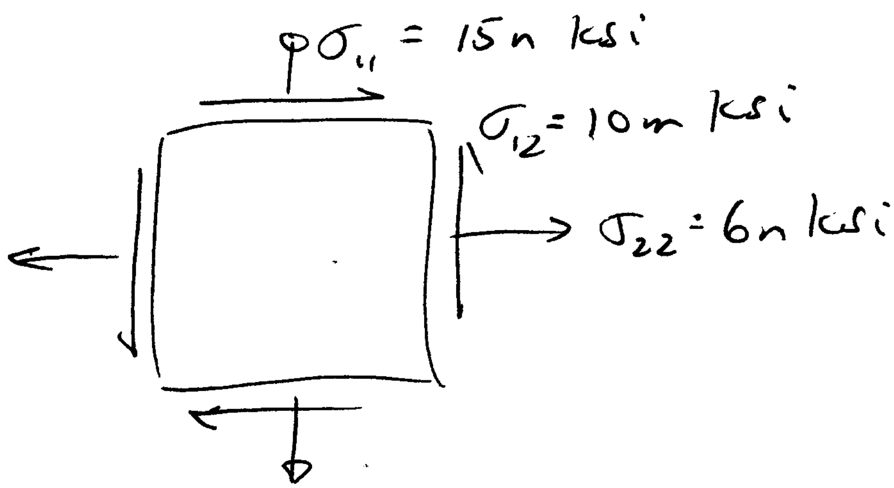
(c) Summarize the results of (b):

critical q [MPa]

Condition	Tresca	von Mises
A	50	57.7
B	60	62.9
C	–	–
D	100	113.4

For each of the conditions, the Tresca criterion gives a more conservative estimate of the yielding characteristic q. The one case where this is not true is condition C where there is no yielding since this is a state of hydrostatic stress.

14.3 Airplane wing skin



$\sigma_{yield} = 50$ ksi

(a) The Tresca Criterion is that:

$$|\sigma_{I} - \sigma_{II}| = \sigma_Y$$
or
$$|\sigma_{II} - \sigma_{III}| = \sigma_Y$$
or
$$|\sigma_{III} - \sigma_{I}| = \sigma_Y$$

Here we have plane stress with $\sigma_{III} = 0$, so this becomes:

$$|\sigma_{I} - \sigma_{II}| = \sigma_Y$$
or
$$|\sigma_{II}| = \sigma_Y \quad \text{or} \quad |\sigma_{I}| = \sigma_Y$$

It is necessary to find the principal stresses for the plane stress case.

Use:
$$\tau^2 - \tau(\sigma_{11} + \sigma_{22}) + (\sigma_{11}\sigma_{22} - \sigma_{12}^2) = 0$$

$$\Rightarrow \tau^2 - \tau(15n + 6n) + [(15n)(6n) - (10m)^2] = 0$$

stresses in [ksi]

Simplify:
$$\tau^2 - 21n\,\tau + 90n^2 - 100m^2 = 0$$

To find the roots, use the quadratic solution:
$$\tau = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \sigma_I, \sigma_{II} = \frac{1}{2}\left\{21n \pm \left[(-21n)^2 - 360n^2 + 400m^2\right]\right\}^{1/2}$$

in [ksi]
$$= 10.5n \pm \frac{1}{2}\left[81n^2 + 400m^2\right]^{1/2}$$

$$\Rightarrow \sigma_I = 10.5n + \frac{1}{2}(81n^2 + 400m^2)^{1/2}$$
$$\sigma_{II} = 10.5n - \frac{1}{2}(81n^2 + 400m^2)^{1/2}$$

The Tresca condition can be rewritten using these expressions:

$$50\text{ ksi} = |\sigma_I - \sigma_{II}| = |(81n^2 + 400m^2)^{1/2}| \quad (1)$$

$$50\text{ ksi} = |\sigma_I| = |10.5n + \frac{1}{2}(81n^2 + 400m^2)^{1/2}| \quad (2)$$

$$50\,ksi = |\sigma_{II}| = |10.5n - \tfrac{1}{2}(81n^2 + 400m^2)^{1/2}| \quad (3)$$

for n and $m \geq 0$, it is always the case that:

- $\sigma_I > \sigma_{II}$
- $\sigma_I > 0$

so only (1) and (2) are operative and without need for $|\ |$

For (1) or (2), express m in terms of n: (note [ksi] is included in all terms)

for (1)....

$$50 = (81n^2 + 400m^2)^{1/2}$$

$$\Rightarrow 2500 - 81n^2 = 400m^2$$

$$\Rightarrow m = \sqrt{6.25 - 0.2025n^2} \quad (1')$$

This limit on n is where

$$6.25 - 0.2025n^2 \geq 0$$

so must have: $0.2025n^2 \leq 6.25$

$$\Rightarrow n \leq 5.55$$

for (2) ....

$$50 = 10.5n + \tfrac{1}{2}(81n^2 + 400m^2)^{1/2}$$

$$\Rightarrow (100 - 21n)^2 = 81n^2 + 400m^2$$

fixing:

$$m = \frac{1}{20}\sqrt{360n^2 - 4200n + 10,000}$$

$$\Rightarrow m = \sqrt{0.9n^2 - 10.5n + 25} \qquad (2')$$

find point where $0.9n^2 - 10.5n + 25 = 0$

lowest + value is maximum allowable

solve via quadratic: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow \frac{+10.5 \pm \sqrt{(10.5)^2 - 4(0.9)(25)}}{2(0.9)}$$

$$= \frac{10.5 \pm \sqrt{110.25 - 90}}{1.8}$$

$$= \frac{10.5 \pm 4.5}{1.8}$$

lowest value = 3.33

$$\Rightarrow n \leq 3.33$$

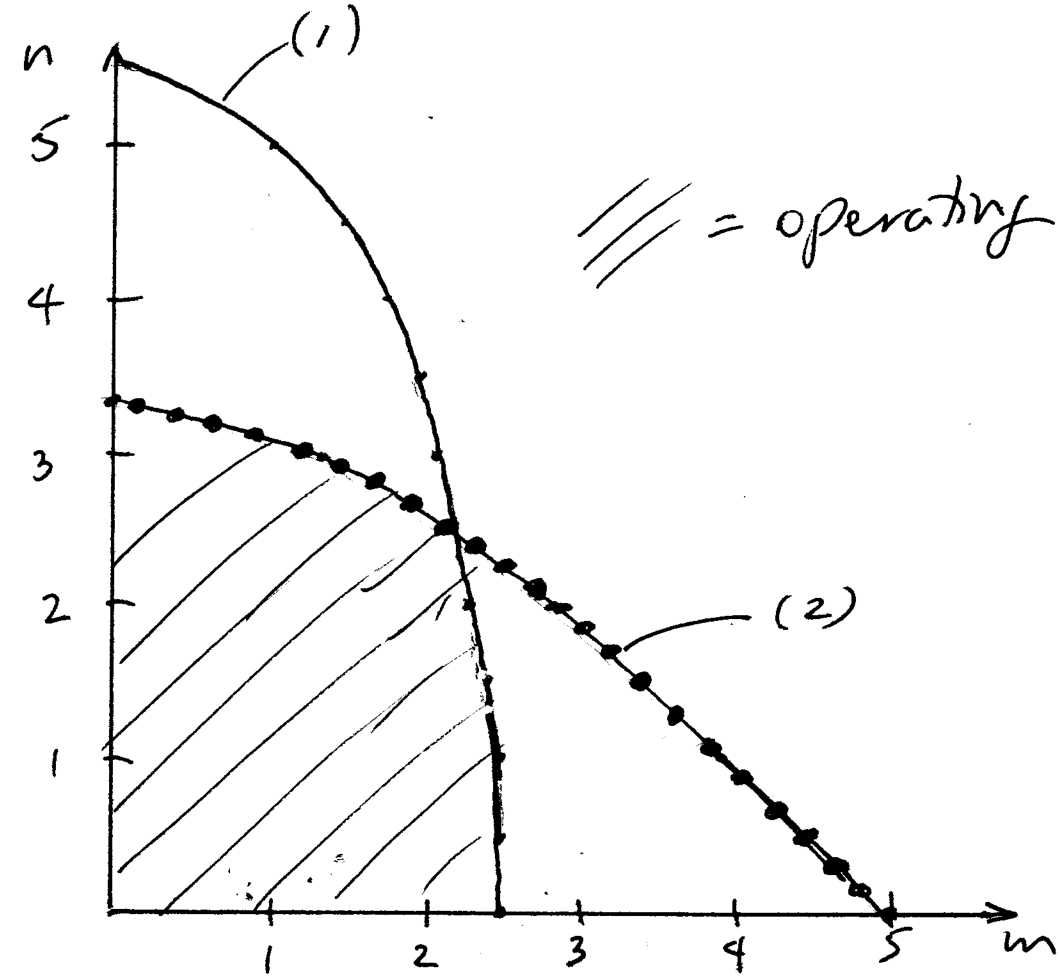
Now use values of n to determine values of m for the two conditions:

n	m via (1')	m via (2')	(3) always less
0	2.5	5	
0.5	2.5	4.5	
1.0	2.5	3.9	
1.5	2.4	3.4	
2.0	2.3	2.8	
2.5	2.2	2.1	
3.0	2.1	1.3	
3.33	2.0	0	
3.5	1.9	for < 0	
5.55	0	X	

Plot:
"Operating envelope" via Tresca condition

(next page)

Operating Envelope
via Tresca Criterion



(b) With the "damage tolerant" approach, use the basic fracture mechanics equation:

$$\sigma_f = \frac{K_c}{\sqrt{\pi a}}$$

Here $2a = 0.40\text{ in} \Rightarrow a = 0.20\text{ in}$

$K_{Ic} = 31\ \text{ksi}\sqrt{\text{in}}$ for ~~the~~ 2024 aluminum

so that:

$$\sigma_f = \frac{31\ \text{ksi}\sqrt{\text{in}}}{\sqrt{\pi(0.20\text{ in})}}$$

$$\Rightarrow \sigma_f = 39.1\ \text{ksi}$$

Thus, if the stress perpendicular to the crack exceeds 39.1 ksi, there is failure. However, the crack could be oriented in any direction, so one must find the principal stresses (i.e. the maximum extensional stresses) and then the related direction for the worst case.

In part (a), found the principal stresses to be:

$$\sigma_I = 10.5n + \frac{1}{2}(81n^2 + 400m^2)^{1/2}$$

$$\sigma_{II} = 10.5n - \frac{1}{2}(81n^2 + 400m^2)^{1/2}$$

with n and m ≥ 0, it was also determined that $\sigma_I > \sigma_{II}$. So consider only σ_I.

The limit condition gives:

$$\sigma_I = 10.5n + \frac{1}{2}(81n^2 + 400m^2)^{1/2} \leq 39.1 ksi$$

[ksi] included in all terms

So boundary determined via:

$$10.5n + \frac{1}{2}(81n^2 + 400m^2)^{1/2} = 39.1 ksi$$

So:

$$81n^2 + 400m^2 = (78.2 - 21n)^2$$

$$\Rightarrow m = \frac{1}{20}\sqrt{360n^2 - 3284n + 6115}$$

$$\Rightarrow m = \sqrt{0.9n^2 - 8.21n + 15.3}$$

A table gives:

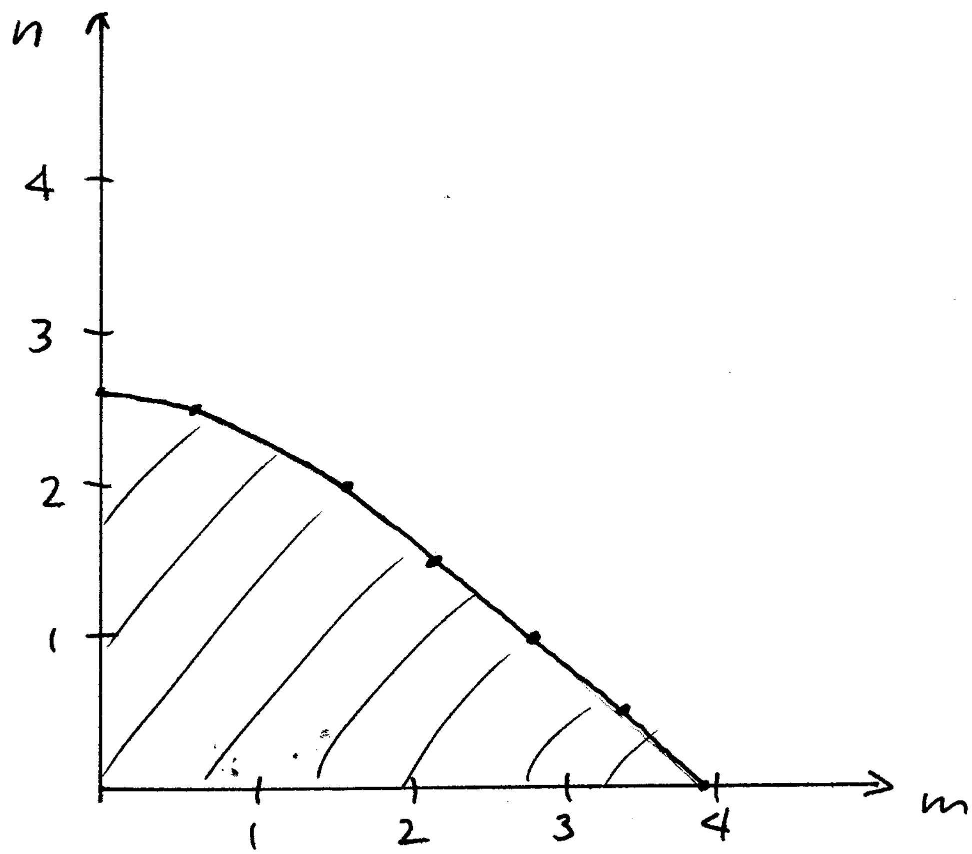
n	0	0.5	1.0	1.5	2.0	2.5
m	3.9	3.4	2.8	2.2	1.6	0.6

limit occurs for $\dfrac{8.21 \pm \sqrt{(8.21)^2 - 4(0.9)(15.3)}}{2(0.9)}$

$$= \frac{4.56 - \sqrt{67.4 - 55.1}}{1.8} = 2.61$$

Plot....

Operating Envelope
v.a Damage Tolerant
Approach



(c) Each of these approaches is different in the criteria used and thus the plots are substantially different. For this particular application, the Tresca condition and the damage tolerant condition are both dependent on the same value of the principal stress and that line has similar shape though different values.

In general, there will be no overall similarity between two such different criteria.

m 14.4

1. D
2. G
3. A
4. H
5. B
6. F
7. E
8. C